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Abstract

The fluctuating field of a jet excited by transient mass injection is simulated numerically. The model is developed by expanding the state vector as a mean state plus a fluctuating state. Nonlinear terms are not neglected and the effect of nonlinearity is studied. The results show a significant spectral broadening in the flow field due to the nonlinearity. In addition, large scale structures are broken down into smaller scales.

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Introduction

A numerical method for simulating the axisymmetric fluctuating field of a forced circular jet has been developed. The method is based on a solution of the Euler equations for homentropic flow in cylindrical coordinates. The jet is forced by transient point mass injection. The source strength is scaled by a parameter ε and the state vector $W = (p,\rho,u,v)$ is expanded as a mean state plus a fluctuating state of order ε. Here p is the pressure, ρ the density and u and v are the axial and radial velocity components respectively.

The solution is expanded in the form $\bar{p} = p_0 + \varepsilon p$, etc. where the mean state (with subscript "0") is assumed to satisfy the unforced Euler equations. The variables (p,ρ,u,v) will represent the fluctuating field in response to the jet forcing. New variables $e = (\rho_0+\varepsilon\rho)u$ and $f = (\rho_0+\varepsilon\rho)v$ are introduced for simplicity. With cylindrical spatial coordinates z(axial) and r(radial) the Euler equations for the fluctuating field become

$$\begin{aligned}
&\rho_t + (\rho U_0+\rho_0 u)_z + (\rho V_0+\rho_0 v)_r + \frac{(\rho V_0+\rho_0 v)}{r} = m \\
&e_t + (e(U_0+\varepsilon u))_z + (e(V_0+\varepsilon v))_r + p_z + fU_{0r} - eV_{0r} - \frac{\varepsilon fu}{r} = \varepsilon mu \qquad (1)\\
&f_t + (f(U_0+\varepsilon u))_z + (f(V_0+\varepsilon v))_r + p_r + eU_{0z} - fV_{0z} - \frac{\varepsilon fv}{r} = \varepsilon mv.
\end{aligned}$$

In the development of system (1) it is assumed that the derivatives of the mean pressure and density in space can be neglected. It is an exact consequence of the full Euler equations with a source of mass injection, and of the expansion into mean and fluctuating states. The source is εm (units density/time). The mean velocities U_0 and V_0 are taken from measurements of a spreading jet. The system is solved in a cylindrical rectangle including a semi-infinite pipe from which the jet exits. The computational domain is illustrated in Fig. 1.

The use of the quasi-momentum variables e and f gives a system from which the fluctuating field can be computed directly rather than as a small part of the total field. In addition the linear limit can be recovered by simply setting $\varepsilon = 0$ in (1). The nonlinear terms are explicitly exhibited. The fluctuating pressure p is obtained from the density ρ by the homentropic relation

$$\bar{p} = A\bar{\rho}^{\gamma} = p_0\left(1 + \varepsilon \frac{\rho}{\rho_0}\right)^{\gamma}, \qquad (2)$$

where $\gamma = 1.4$ in air.

The fundamental assumption entering into the derivation of (1) is that the mean state is a solution to the unforced Euler equations. This is not true for a state determined from experimental measurements. Numerical experiments have verified however, that the qualitative features of the fluctuating solution are insensitive to small changes in the mean state and thus we believe that the solution to (1) qualitatively represents the fluctuating field in response to a a given source.

The source is assumed to be a delta function in space (modelled by a sharp Gaussian) with a pulse-like time dependence. The source location, z_s, is on the jet centerline approximately 1.2 jet diameters downstream of the nozzle exit. Specifically

$$m(t,z,r) = f(t)\delta\left(r^2+(z-z_s)^2\right), \tag{3}$$

where the function $f(t)$ is

$$f(t) = e^{-(at^2+bt^{-2})}, \tag{4}$$

for suitable constants a and b. The use of (4) permits the investigation of a broad band spectrum.

The fluctuating field described by (1) reduces to the acoustic field for large distances. The near field and flow field are dominated by instability waves which are generated because the mean flow is linearly unstable. The pulse is assumed to dominate the natural sources of jet noise. These natural sources are both the linear and nonlinear terms in (1). In real jet these terms are determined from the turbulent fluctuations whereas in the numerical model these natural sources are excited by the pulse. The important physical effect is the generation of packets of instability waves in the flow. Large scale structures which are related to mean flow instabilities have been observed experimentally in both forced and unforced jets [1,2]. These structures interact with and modify the resulting acoustic fields. The model permits this interaction to be studied in both the linear ($\varepsilon = 0$) and nonlinear ($\varepsilon \neq 0$) regimes. Calculations with the linear model and comparison with experiments are described elsewhere [3,4]. In the rest of this paper we describe the numerical requirements in order to compute with this model and the effects of the nonlinear terms on the fluctuating field.

Numerical Model

In order to numerically integrate (1) it is necessary to resolve the solution over large length scales (far field, near field, and flow field). This necessitates the use of a higher order difference scheme. The system can be written in the form

$$w_t + F_z + G_r = H, \tag{5}$$

where w is the vector (ρ,e,f) and F, G, and H are appropriate functions. Equation (5) is split into two one-dimensional operators in z and r. Each 1-d system is integrated by using a fourth order version of the MacCormack scheme [5]

$$\begin{aligned} \tilde{w}_i^{n+1} &= w_i^n + \frac{\Delta t}{6\Delta x}\left(-7F_i+8F_{i+1}-F_{i-2}\right) + \Delta t H_i \\ w_i^{n+1} &= 1/2\left(\tilde{w}_i^{n+1} + w_i^n + \frac{\Delta t}{6\Delta x}\left(7\tilde{F}_i-8\tilde{F}_{i-1}+\tilde{F}_{i-2}\right) + \Delta t\tilde{H}_i\right), \end{aligned} \tag{6}$$

together with a symmetric variant (H_i is obtained from some splitting of H). Typical grids require of the order of 40,000 grid points over distances of the order of 50 jet diameters in all directions. Our experience has been that on problems of this size, second order schemes are not sufficient to obtain accurate solutions with the amount of resolution that is feasible. The explicit scheme (6) naturally lends itself to vectorization and has been implemented on the CDC CYBER-203 with great efficiencies.

In addition it is necessary to impose boundary conditions which accurately simulate outgoing radiation at the far field boundaries. A family of radiation boundary conditions has been developed which provide increasingly accurate approximations to outgoing radiation. The leading member of this family is

$$p_t + \rho_\infty c_\infty \tilde{u}_t + \frac{p}{d} = 0, \tag{7}$$

where c_∞ is the ambient sound speed and ρ_∞ the ambient density. Here $d^2 = r^2 + z^2$ and $\tilde{u}$ is the outgoing radial velocity based on a spherical coordinate system near the source m.

System (1) includes terms proportional to r^{-1}. This singularity at the axis is resolved by including these terms in the flux vector G when r=0. In addition it is necessary to modify the difference formula (6) at boundaries. This is accomplished by introducing fictitious grid points outside of the computational domain and using a third order extrapolation of the flux function (F or G). This approach was found to be the most readily vectorizable. It has been verified that the resulting scheme is fourth order accurate.

Nonlinear Results

We next describe results illustrating the effect of the nonlinear terms on the fluctuating field. In Figs. 2 and 3 the fluctuating pressure is shown as a function of axial location z/D (D is the jet diameter) and non-dimensional time tc_∞/D for two different radial positions and for $\varepsilon = 0.00$ and $\varepsilon = 0.05$. All figures show an acoustic wave (speed of sound normalized to unity) in the downstream direction trailed by several much larger waves. These are instability waves which travel with a speed of approximately $.7U_j$ where $U_j = .66c_\infty$ is the jet exit velocity. A series of acoustic ripples can also be seen propagating upstream. These are due to diffraction of the upstream acoustic wave by the nozzle lip.

The figures indicate that the nonlinear terms have little effect on the primary acoustic pulse and on the acoustic diffraction from the nozzle lip. The nonlinearity has a pronounced effect on the instability waves. Increasing the nonlinearity causes these predominantly large scale structures to break up into smaller scale structures. This can be seen in both the additional ripples which trail the instability waves and a sharpening of the individual pulses indicating an enhanced high frequency content. It is also evident that for increasing r/D these smaller scale structures are comparable in amplitude to the primary instability waves.

In Figs. 4 and 5 the fluctuating vorticity is shown for two fixed times and for $\varepsilon = 0.00$ and $\varepsilon = 0.05$. The intense vortices in Fig. 4 correspond to the instability waves in Figs. 2 and 3 while the vortices at the later time in Fig. 5 represent a residual shedding of vorticity from the nozzle lip. It is apparent from the figures that nonlinear effects tend to slow down the vortices as they propagate downstream. Thus the trailing vortices catch up with the leading vortices and a possible pairing of vortices can be observed. True vortex merging, which has been observed experimentally [6], depends heavily on viscosity as well as nonlinearity and is not simulated here.

In Figs. 6a and 6b the normalized power spectral densities (PSD) for the fluctuating axial velocity and pressure are plotted for the linear and nonlinear cases as a function of the Helmholtz number fD/c_∞ (f is the frequency). These figures clearly indicate the shift into higher frequencies and the overall broadening of the spectra caused by the nonlinear effects. Then results are typical for the fluctuating field at all points.

Discussion

The fundamental difference between the nonlinear and linear computations is the breakdown of the large scale structures into smaller scale structures. This is associated with a transfer of energy into higher frequencies or equivalently a broadening of the spectral content of the fluctuating field. It is also illustrated by the increased interaction between the different vortices. In real jets this is a fundamental step in the transition to fully developed turbulence. The results indicate that at least the initial stages of this energy cascade into smaller scales can be simulated just by the nonlinear terms of an axisymmetric and inviscid calculation.

The generation of smaller scale fluctuations fully justifies the use of fourth order spatial discretizations. The numerical scheme is accurate and is in general stable. Higher values of ε can be readily computed although the Gaussian approximation to the δ function source will have to be smoothed out.

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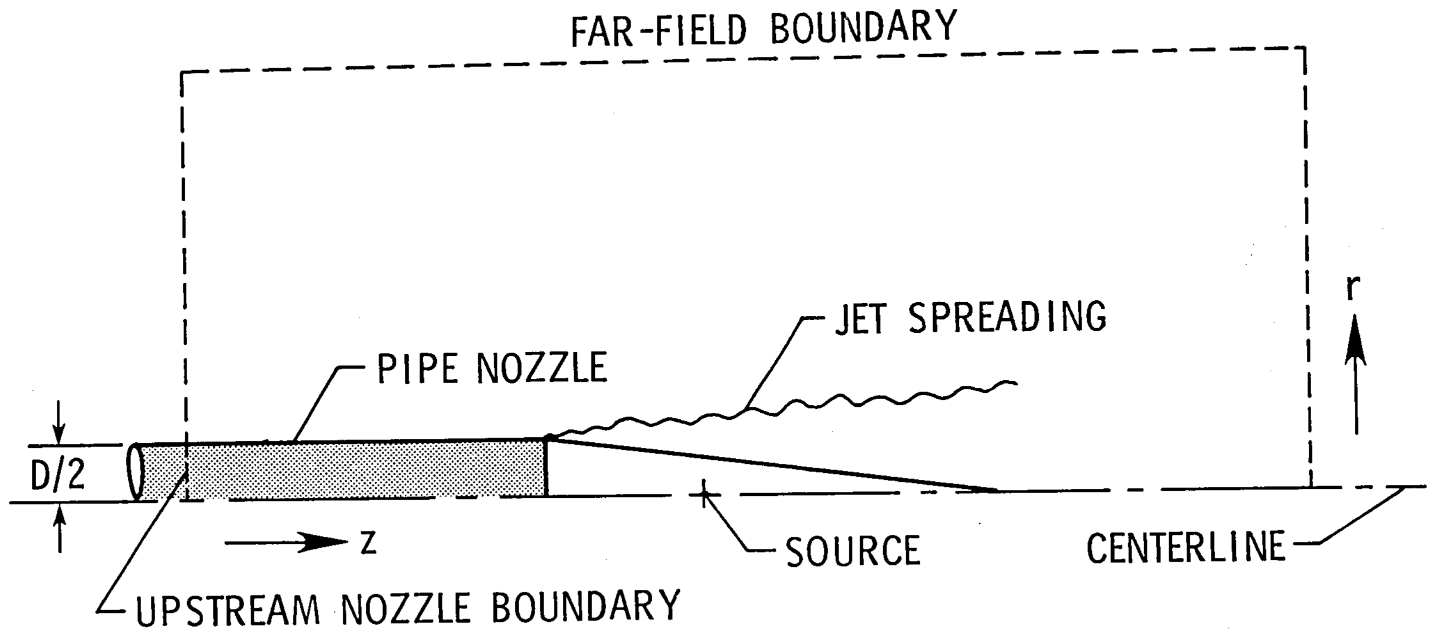

Figure 1. Computational Domain

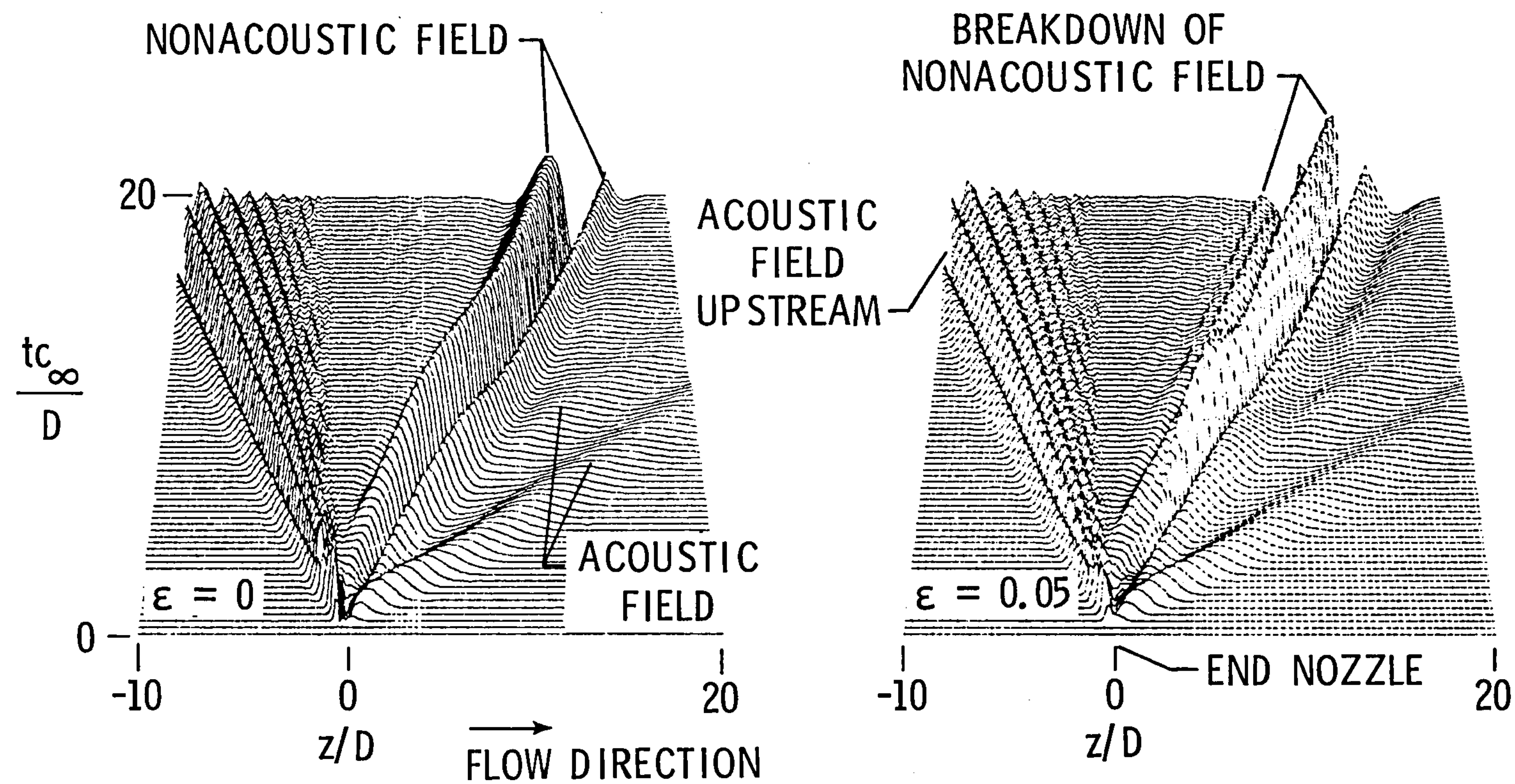

Figure 2. Three-dimensional plots of the fluctuating pressure $r/D = .29$.

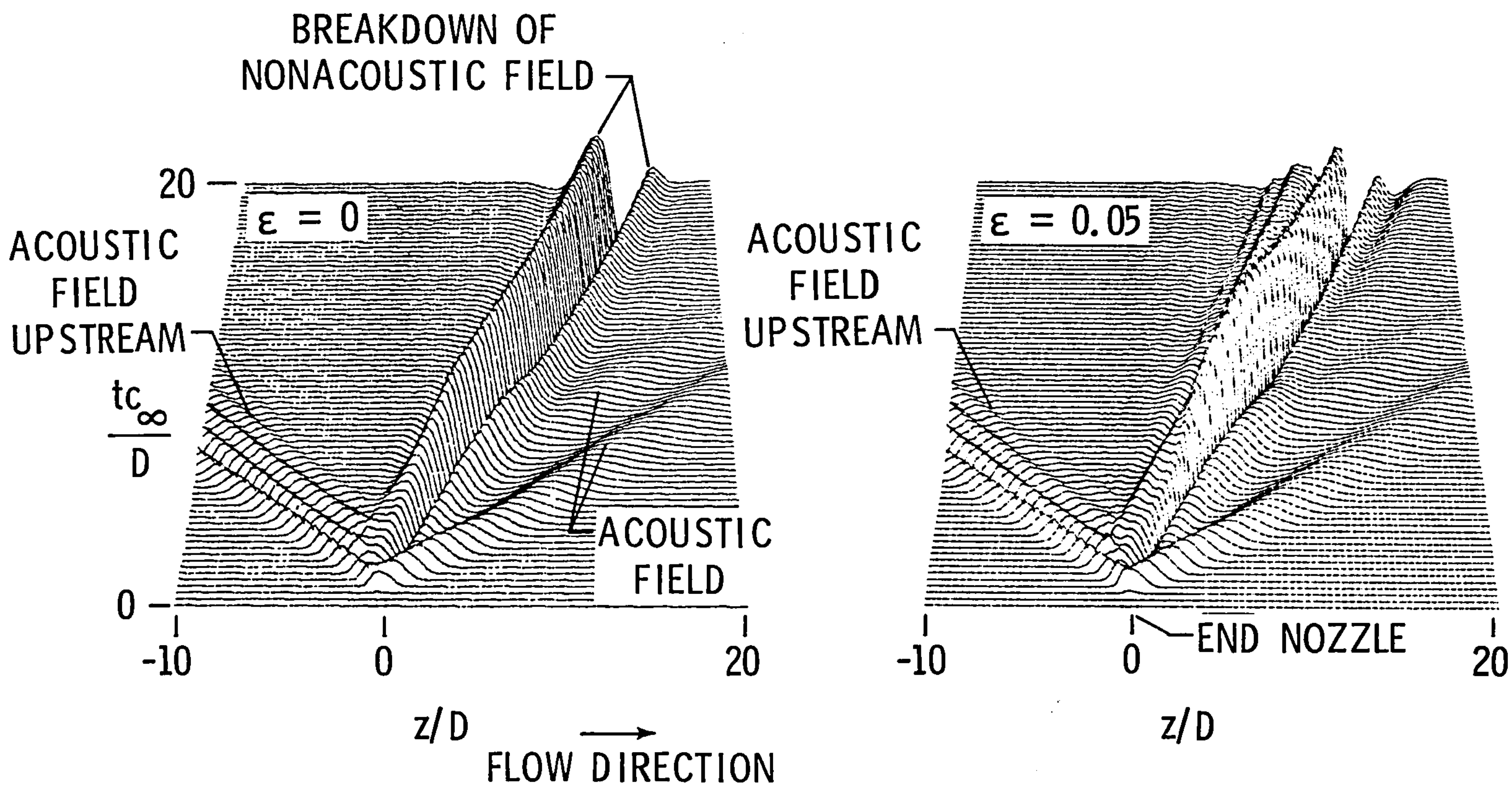

Figure 3. Three-dimensional plots of the fluctuating pressure $r/D = .61$.

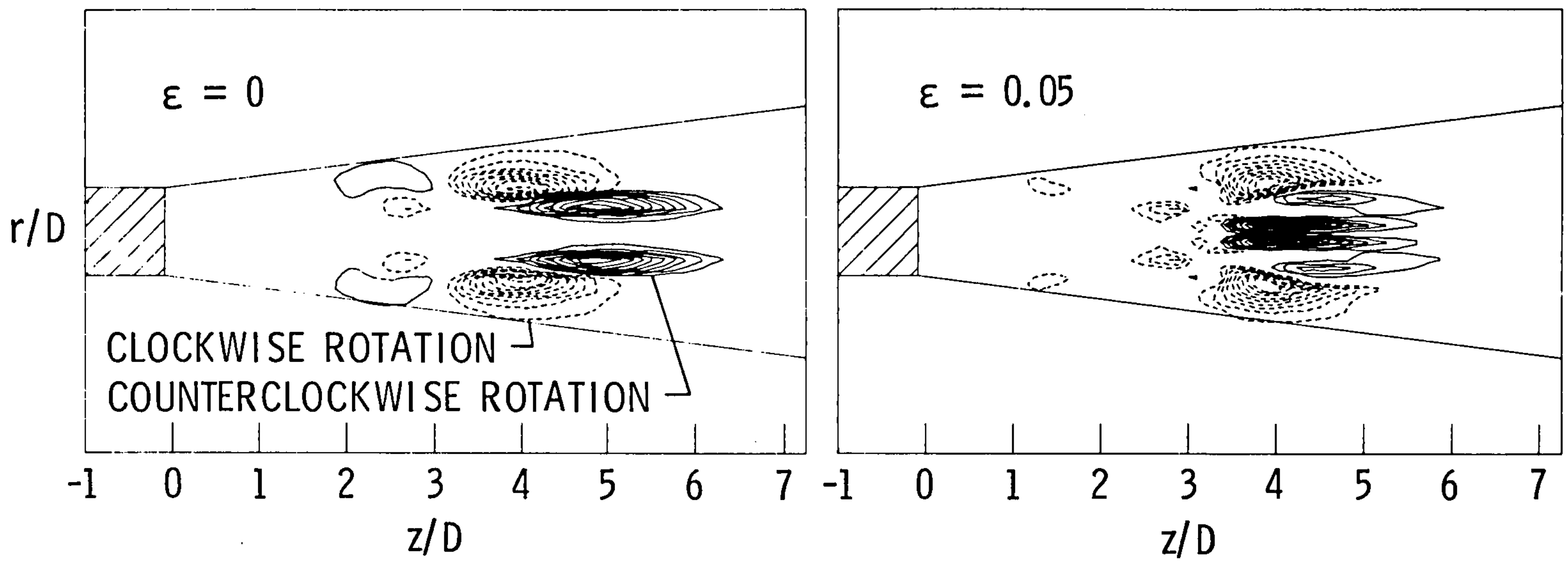

Figure 4. Fluctuating vorticity at $tc_{\infty}/D = 10$.

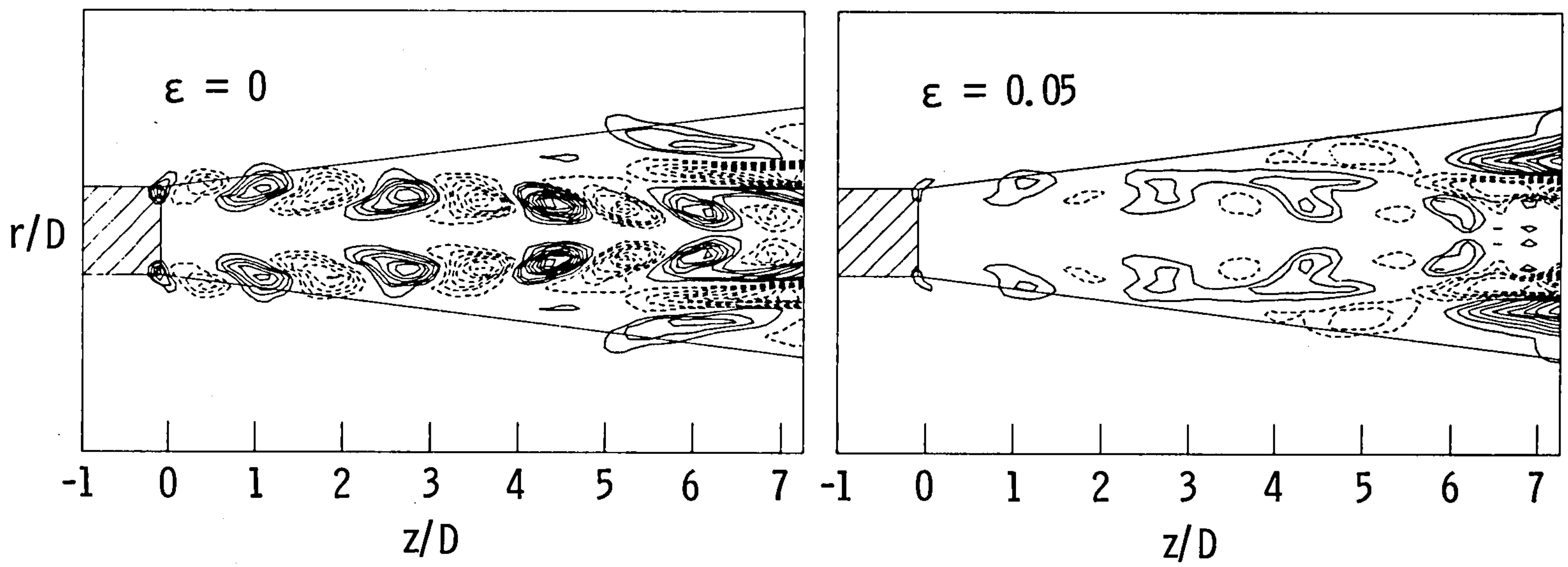

Figure 5. Fluctuating vorticity at $tc_\infty/D = 30$.

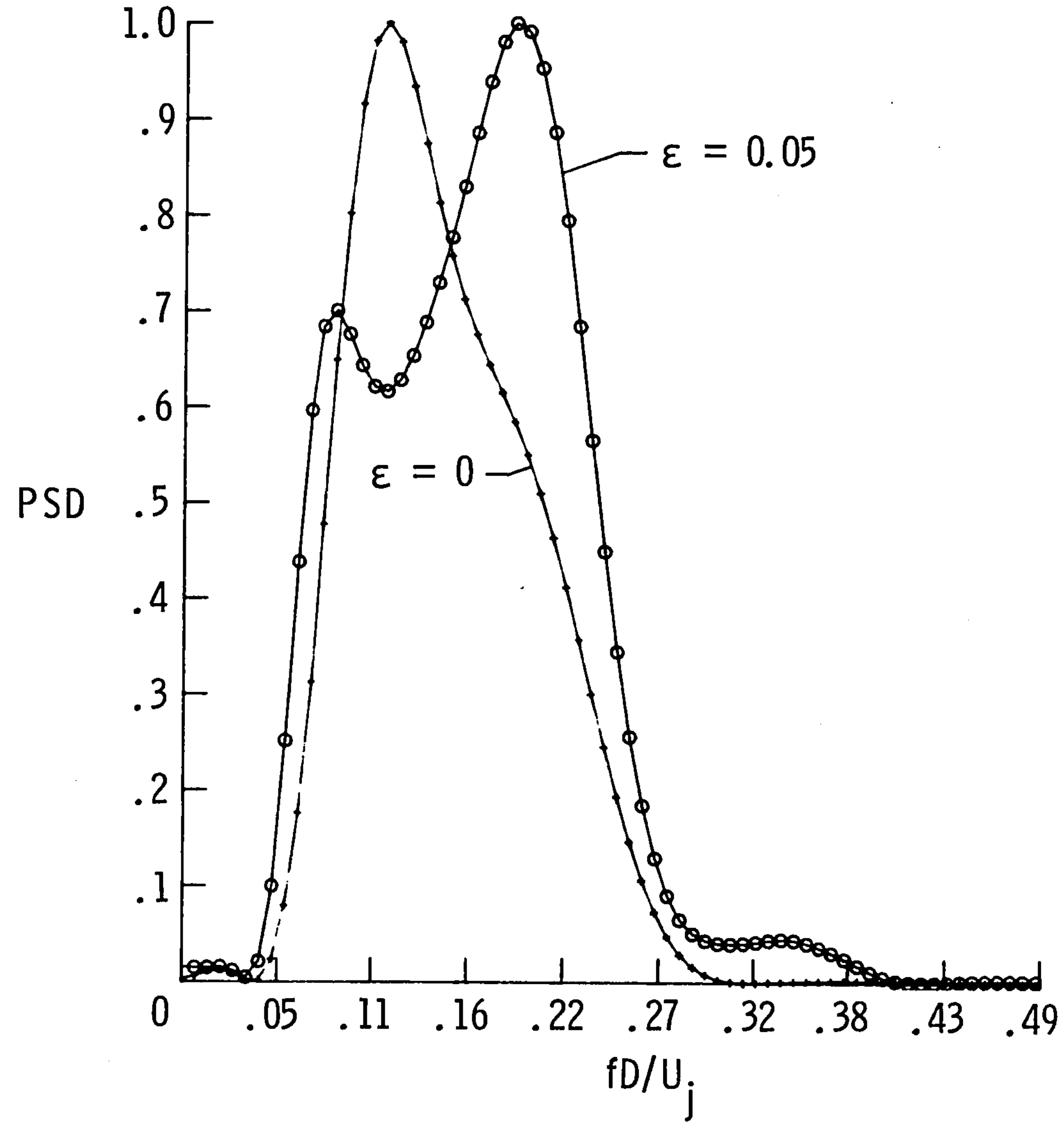

Figure 6a. Flow field velocity spectra z/D = 7.3, r/D = 1.5.

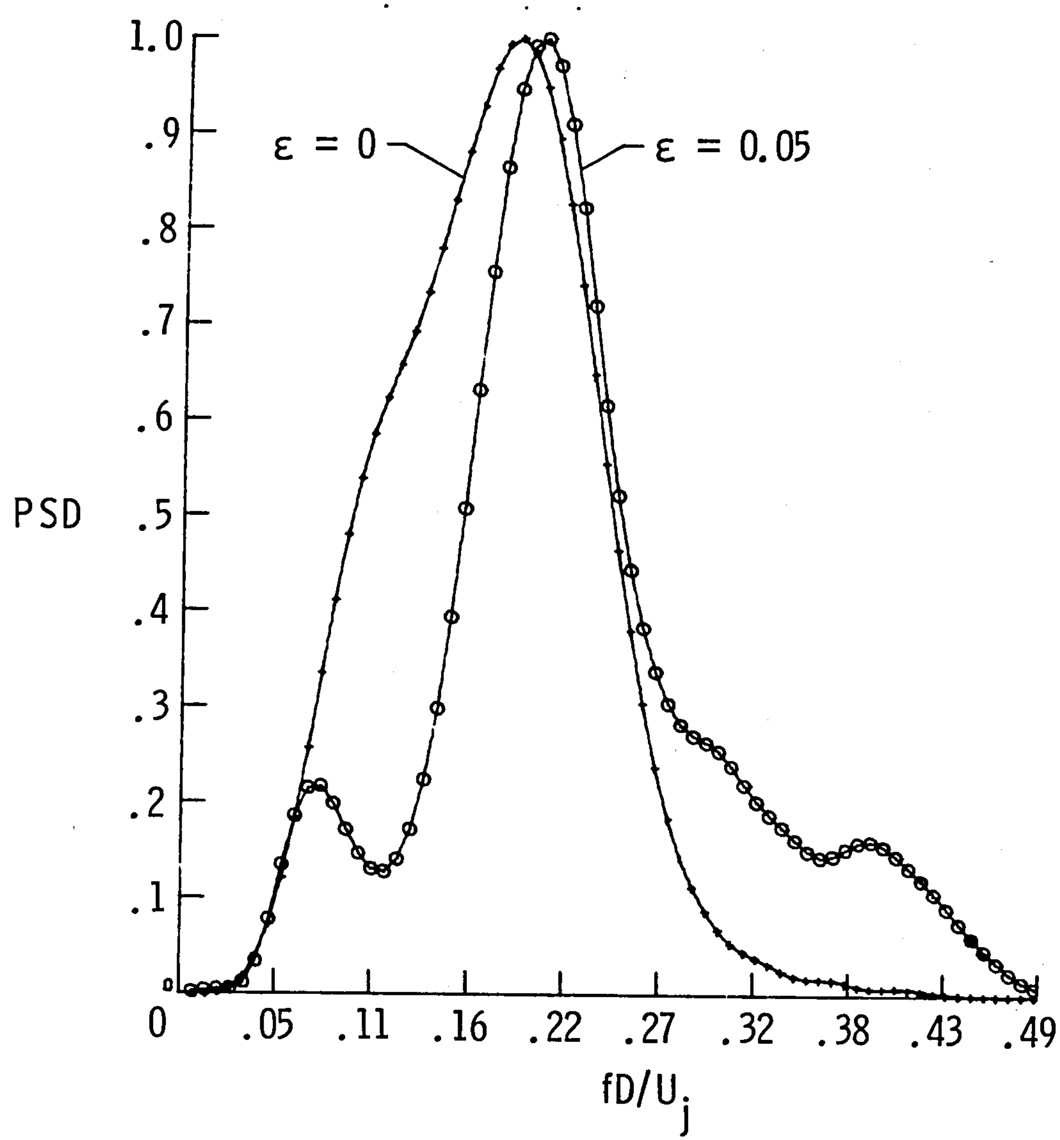

Figure 6b. Acoustic far-field spectra z/D = 38, r/D = 20.

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